

Borel integration prescription suggested in pQCD by infrared renormalons

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Relation between the infrared renormalons and the Borel resummation prescriptions in perturbative QCD (pQCD) is investigated. In principle, there is a large class of allowed solutions reflecting the well-known renormalon ambiguity. However, not all of the solutions can be regarded as natural in the pQCD framework. One specific recently suggested prescription resulted in the Principal Value and an additional power-suppressed correction that is consistent with the Operator Product Expansion. This prescription is investigated here. Arguments requiring the finiteness of the result for any power coefficient of the leading infrared renormalon, and the consistency in the case of the absence of that renormalon, require that this prescription be modified. The apparently most natural modification leads to the result represented solely by the Principal Value.

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QCD observables are quantities which, in general, are known to possess renormalons, i.e., singularities of the Borel transform on the real axis [1]. They appear as a consequence of the specific asymptotic behavior of the high-order perturbation coefficients. The singularities on the positive axis, called infrared (IR) renormalons, represent an obstacle to the Borel integration and lead to the well-known renormalon-induced ambiguity for the observable. The most widely used prescription for fixing this ambiguity has been to perform the Borel integration parallel to the positive real axis and taking the real part of the resulting integral (Principal Value; see, e.g., Refs. [2, 3, 4, 5, 6]). Arguments based on analyticity considerations in the momentum plane [7] and in the coupling parameter plane [8] were shown to favor this prescription. This prescription has been chosen apparently because its simplicity makes it mathematically attractive. The additional terms, called nonperturbative terms, are then expected to have the power-suppressed form of the higher-twist terms in the Operator Product Expansion (OPE), and the perturbative QCD (pQCD) is usually expected to be unable to predict the strength of such terms.¹

Recently, a specific prescription, based on the IR renormalon considerations, has been proposed [9] to fix the strength of such higher-twist terms, and this method gave encouraging numerical results in the case of the resummation of the Gross–Lewellyn-Smith sum rule [10] and of the heavy-quark potential [11]. The main observation was that the IR renormalon induces in the Borel-integrated quantity a nonphysical cut along the positive axis in the complex plane of the coupling parameter z , and that this cut structure can be naturally eliminated by subtracting a cut function proportional to $(-z)^\nu$ where ν is related with the power coefficient of the renormalon singularity. Further, the energy dependence of the subtraction term, when the coupling parameter z is positive, is consistent with the predictions of the OPE for the corresponding higher-twist term. For positive z , the result is the Principal Value of the Borel integration minus the aforementioned term. A somewhat speculative interpretation of this result suggests that in this way the genuine nonperturbative higher-twist effect is obtained (the correction term to the Principal Value), although the method is based on perturbative (pQCD + renormalons) knowledge only. A more conservative interpretation would be that this result represents “the most that we can get” out of pQCD, i.e., the “natural” basis to which one should eventually add other contributions to the aforementioned higher-twist term; such genuine nonperturbative contributions would involve the vacuum expectation values of the higher-twist operators appearing in the OPE.

In the present work, the aforementioned method is scrutinized, and the results of this investigation give, as a by-product, more support to the second of the two mentioned interpretations of the pQCD renormalon resummation results. Even more so, the results suggest that the mentioned “natural” resummed perturbative contribution is just the Principal Value of the Borel integral, and not the expression of the method suggested in Ref. [9].

Let us consider a Euclidean QCD observable $\Delta[a(Q)]$ where the quark mass effects are neglected. Therefore, it can be regarded as depending on the energy $Q \equiv \sqrt{-q^2}$ of the corresponding process only via the QCD coupling parameter $a(Q) \equiv \alpha_s(Q; \overline{MS})/\pi$ whose running is determined by the (\overline{MS}) renormalization group equation

$$\frac{\partial a(\mu)}{\partial \ln \mu^2} = -\beta_0 a^2 (1 + c_1 a + c_2 a^2 + c_3 a^3 + \dots). \quad (1)$$

For simplicity of argument, we will consider only the effects of the leading IR renormalon and will neglect the

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¹ The IR renormalons predict the energy dependence of such terms.

subleading IR renormalons.

First, let us review the method of Ref. [9] in detail. The presentation of the method is here somewhat different from that of Ref. [9]. The Borel transform $B(b)$ of $\Delta[a(Q)]$ has a singularity at $b \geq n$ ($n = 1, 2, 3, \dots$) of the form

$$B(b) = \frac{C}{(1-b/n)^{1+\nu}} [1 + \kappa_1 (1-b/n) + \kappa_2 (1-b/n)^2 \dots] + \text{analytic part} . \quad (2)$$

Here $\nu = (nc_1 - \gamma_{2n})/\beta_0$, where γ_{2n} is the one-loop coefficient of the anomalous dimension of the corresponding higher-twist operator (with dimension $d=2n$) in the OPE for $\Delta[a(Q)]$. For definiteness, the renormalization scale μ is taken to be $\mu = Q$, and the renormalization scheme is also regarded as fixed, e.g., $\overline{\text{MS}}$. The Borel integral $\Delta_{\text{BI}}(z)$ for the complex coupling $z \equiv \beta_0 a(Q)/n = |z| \exp(i\phi)$ is defined as

$$\Delta_{\text{BI}}(z) = \frac{1}{\beta_0} \int_0^{+\infty \exp(i\phi)} db \exp\left(-\frac{b}{nz}\right) B(b) , \quad (z = |z|e^{i\phi}, b = |b|e^{i\phi}) . \quad (3)$$

For z near the positive real axis $z = |z| \pm i\varepsilon$ [$|z| = \beta_0 a(Q)/n > 0$], it is straightforward to show from (3), with the help of the Cauchy theorem, the following formula:

$$\begin{aligned} \Delta_{\text{BI}}(z)|_{z=|z|\pm i\varepsilon} &= \frac{1}{\beta_0} \int_{\pm i\varepsilon}^{+\infty \pm i\varepsilon} db \exp\left(-\frac{b}{n|z|}\right) B(b) \\ &= \frac{1}{\beta_0} \int_{\pm i\varepsilon}^{+\infty \pm i\varepsilon} db \exp\left[-\frac{b}{\beta_0 a(Q)}\right] B(b) \quad (\varepsilon \rightarrow +0) . \end{aligned} \quad (4)$$

The real part of this is the Principal Value. As a consequence of the IR singularity (2) at $b \geq n$, the integral (3) has a discontinuity (cut) at the positive real axis $z \geq 0$. Namely, the quantity (4) is not real and its discontinuity shows up in its imaginary part. This can be seen by introducing in the integrand of Eq. (4) the new complex integration variable t via $b = n(1 + |z|t)$

$$\begin{aligned} \text{Im}\Delta_{\text{BI}}(z = |z|\pm i\varepsilon) &= \pm \frac{1}{2i\beta_0} \int_{\mathcal{C}_t} db \exp\left(-\frac{b}{n|z|}\right) \frac{C}{(1-b/n)^{1+\nu}} \left[1 + \sum_{m=1}^{\infty} \kappa_m (1-b/n)^m + \dots\right] \\ &= \mp C \frac{n}{\beta_0} e^{-1/|z|} \sin(\pi\nu) \left[|z|^{-\nu} \Gamma(-\nu) + \sum_{m=1}^{\infty} \kappa_m (-1)^m |z|^{-\nu+m} \Gamma(-\nu+m)\right] . \end{aligned} \quad (5)$$

The (Hankel) contour \mathcal{C}_t is depicted in Fig. 1, and the expression (5) is obtained from the known Hankel contour form of the Gamma function (see, for example, [12])

$$\Gamma(s) \sin(\pi s) = \frac{\pi}{\Gamma(1-s)} = + \frac{1}{2i} \int_{\mathcal{C}_t} dt e^{-t} (-t)^{-1+s} \quad (|s| < \infty) , \quad (6)$$

in the special cases $s = -\nu, -\nu + 1, \dots$. Because $(-|z| \mp i\varepsilon) = |z| \exp(\mp i\pi)$ and thus $(-|z| \mp i\varepsilon)^{-\nu+m} = |z|^{-\nu+m} (-1)^m [\cos(\pi\nu) \pm i \sin(\pi\nu)]$, Eq. (5) leads to

$$\Delta_{\text{BI}}(z) = -C \frac{n}{\beta_0} e^{-1/z} \left[(-z)^{-\nu} \Gamma(-\nu) + \sum_{m=1}^{\infty} \kappa_m (-z)^{-\nu+m} \Gamma(-\nu+m) \right] + \tilde{\Delta}_{\text{BI}}(z) , \quad (7)$$

where $\tilde{\Delta}_{\text{BI}}(z)$ is a function without cuts in the complex z -plane since the first expression on the right-hand side absorbs the renormalon-induced cut (5) at $z \geq 0$. The first expression, when $z = |z| \pm i\varepsilon = \beta_0 a(Q)/n \pm i\varepsilon$ is at the real positive axis, has the Q -dependence $Q^{-2n} \alpha_s(Q)^{\gamma_{2n}/\beta_0} [1 + \mathcal{O}(\alpha_s)]$, the same as the corresponding power-suppressed (higher-twist) term $\langle \mathcal{O}_{2n} \rangle^{(Q)} / (Q^2)^n$ in the OPE. The imaginary part of this expression, i.e., expression (5), must be identified as the imaginary part of the contribution from the leading IR renormalon (2). The central assumption of the method is that the full first expression on the right-hand side of (7), which contains the full nonphysical cut (5) at $z > 0$, represents the nonphysical cut-function which is to be eliminated

$$\Delta_{(\text{cut1})}(z) = -C \frac{n}{\beta_0} e^{-1/z} \left[(-z)^{-\nu} \Gamma(-\nu) + \sum_{m=1}^{\infty} \kappa_m (-z)^{-\nu+m} \Gamma(-\nu+m) \right] . \quad (8)$$

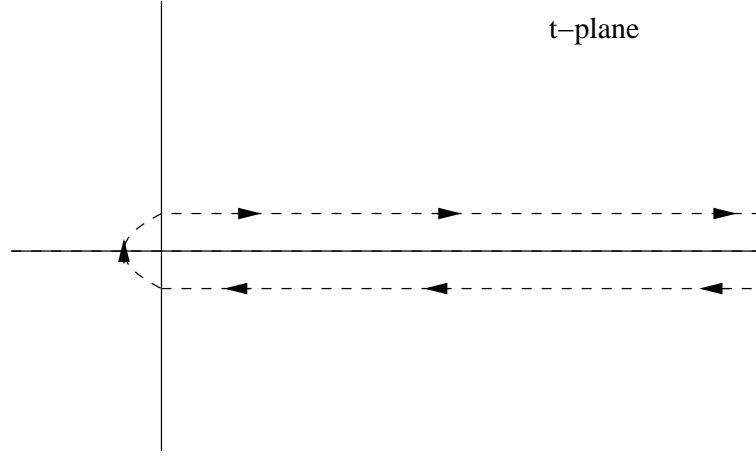


FIG. 1: The path \mathcal{C}_t in the integrals (5) and (6).

This same cut contribution, for $z = |z| \pm i\varepsilon$ at the positive real axis, can be obtained also by Borel-integrating the nonanalytic part of the Borel transform (2) along its cut $b > n$ above or below the real axis in analogy with the full Borel-integrated quantity (4). To show this, we use the new real integration variable t such that $b = n(1+zt)$, and therefore $b > n \pm i\varepsilon$ corresponds to $t > 0$

$$\Delta_{\text{BI}}^{(\text{nonan.})}(z = |z| \pm i\varepsilon; b > n) = \frac{1}{\beta_0} \int_{n \pm i\varepsilon}^{\infty \pm i\varepsilon} db \exp\left(-\frac{b}{nz}\right) B^{(\text{nonan.})}(b) \quad (9)$$

$$= +C \frac{n}{\beta_0} z e^{-1/z} \int_0^{+\infty} dt e^{-t} (-z)^{-1-\nu} t^{-1-\nu} \left[1 + \sum_{m=1}^{\infty} \kappa_m (-z)^m t^m \right] \quad (10)$$

$$= -C \frac{n}{\beta_0} e^{-1/z} \left[(-z)^{-\nu} \Gamma(-\nu) + \sum_{m=1}^{\infty} \kappa_m (-z)^{-\nu+m} \Gamma(-\nu+m) \right]. \quad (11)$$

For z away from the real positive axis, the analytic continuation of this expression in z keeps its form (11) unchanged, i.e., precisely the cut-function (8). The Borel-integration over $t > 0$ ($\Leftrightarrow b > n \pm i\varepsilon$) in the first term of Eq. (10) converges only when $\text{Re}(\nu) < 0$ [in the m 'th term: when $\text{Re}(\nu-m) < 0$], and gives the result (11). When $\text{Re}(\nu) \geq 0$ [or: $\text{Re}(\nu-m) \geq 0$], the result (11) represents the analytic continuation in ν .

The subtraction of the cut-function contribution (8) leads to the final result for the resummed value of the observable at the positive real value $z = |z| = \beta_0 a(Q)/n > 0$

$$\begin{aligned} \Delta[z = \beta_0 a(Q)/n] &= \Delta_{\text{BI}}(|z| \pm i\varepsilon) - \Delta_{(\text{cut1})}(|z| \pm i\varepsilon) = \tilde{\Delta}_{\text{BI}}[z = |z| = \beta_0 a(Q)/n] \\ &= [\text{Re} \mp \cot(\pi\nu) \text{Im}] \int_{\pm i\varepsilon}^{\infty \pm i\varepsilon} db \exp\left[-\frac{b}{\beta_0 a(Q)}\right] \frac{R(b)}{(1-b/n)^{1+\nu}}. \end{aligned} \quad (12)$$

In the Borel integration here, the exactly known IR renormalon singularity has been factored out explicitly. Function $R(b) = (1-b/n)^{1+\nu} B(b)$ (whose truncated perturbation series up to $\sim b^2$ is known exactly in the case of several QCD observables) has a much weaker singularity at $b=n$ than $B(b)$. In Eq. (12), the equality $\text{Re}\Delta_{(\text{cut1})}(|z| \pm i\varepsilon) = \pm \cot(\pi\nu) \text{Im}\Delta_{(\text{cut1})}(|z| \pm i\varepsilon)$ was taken into account, a direct consequence of Eqs. (5), (7), and $(-|z| \mp i\varepsilon)^{-\nu+m} = |z|^{-\nu+m} (-1)^m [\cos(\pi\nu) \pm i \sin(\pi\nu)]$.

The renormalon power coefficient $\nu = (nc_1 - \gamma_{2n})/\beta_0$ should be regarded as a general parameter which can take on, in principle, any (real) value. For example, when varying the number of effectively active quark flavors n_f continuously, ν changes continuously. Yet another example is given by the large- β_0 approximation, when $c_1 \rightarrow 0$ and $\nu \rightarrow 0$. The residue C of the renormalon in general does not vanish in the large- β_0 limit.

The cut-function (8), as a function of general ν and when the coupling parameter is near the positive real cut axis $z_{\pm} = |z| \pm i\varepsilon = \beta_0 a(q)/n \pm i\varepsilon$, can be rewritten in the following form, by taking into account the first identity in Eq. (6):

$$\Delta_{(\text{cut1})}(z = |z| \pm i\varepsilon) = -C \frac{n}{\beta_0} e^{-1/|z|} [-\pi \cot(\nu\pi) \mp i\pi] \left\{ \frac{|z|^{-\nu}}{\Gamma(1+\nu)} + \sum_{m=1}^{\infty} \kappa_m \frac{|z|^{-\nu+m}}{\Gamma(1+\nu-m)} \right\}. \quad (13)$$

The central assumption of the method, i.e., the subtraction of the cut-function (8) from the Borel-integrated value, appears to be plausible and natural, especially because the cut function (8) [\Leftrightarrow (13)] has a simple form, and because it represents precisely the contribution of the Borel integration of the nonanalytic part of the Borel transform (2) along the cut $b \geq n$ parallel to the real axis, as explained in Eqs. (9)–(11). However, there are at least two problems with this method when we impose on it the plausible condition that it should work for any (real) value of the power coefficient ν .

1. When ν is nonnegative integer ($\nu = k; k = 0, 1, 2, \dots$), the cut-function (13) is infinite, because $\cot(\pi\nu)$ diverges there. Ref. [9] mentions that in such a case the residue C of the renormalon must disappear, making this cut-function finite. This appears to be unlikely, as argued above; in particular, for Adler function in the large- β_0 approximation ($n = 2, c_1 = 0, \gamma_4 = 0$) this would mean the disappearance of the leading IR renormalon. Note that, in contrast to $\cot(\pi\nu)$, the factor $1/\Gamma(1 + \nu)$ in Eq. (13) is an analytic function in the entire complex ν -plane.
2. When ν is a negative integer $\nu = -1, -2, \dots$, the IR renormalon singularity (2) with the cut disappears (even when $C \neq 0$), the Borel transform is analytic. This means that $\Delta_{(\text{cut1})}(z)$ must be zero. However, according to Eq. (13) $\Delta_{(\text{cut1})}(z) \neq 0$ – note: $-\pi \cot(\pi\nu)/\Gamma(1 + \nu) = (-1)^{k+1}(k-1)! \neq 0$ when $\nu = -k = -1, -2, \dots$. Again, it is the poles of $\cot(\pi\nu)$ that cause the problems, but this time at $\nu = -1, -2, \dots$

If we take the view-point that the method should be taken as the starting point nonetheless, because of the mentioned plausibility and naturalness of the choice of (8), we should definitely modify it so that the aforementioned two problematic points are eliminated. This can be done in a natural way, by inspecting again the expression (13). The two problematic aspects arose because of the poles of the factor $\cot(\pi\nu)$, at $\nu = 0, \pm 1, \pm 2$, etc. The function $f(\nu) = \cot(\pi\nu)$ should thus be regularized in order to eliminate both problems. The apparently most natural regularization, i.e., elimination of the poles, is obtained by subtracting the simple pole functions with the coefficients being equal to the residues of $f(\nu)$ at those poles. This gives:

$$-\pi \cot(\pi\nu) \mapsto -\pi \cot(\pi\nu) + \frac{1}{\nu} + \sum_{k=1}^{\infty} \left(\frac{1}{\nu - k} + \frac{1}{\nu + k} \right) \quad (14)$$

$$= -\pi \cot(\pi\nu) + \frac{1}{\nu} + 2\nu \sum_{k=1}^{\infty} \frac{1}{\nu^2 - k^2} \equiv 0 \quad (15)$$

We see that the function $\cot(\pi\nu)$ is identical to the sum of the corresponding simple pole functions. Only some of the meromorphic functions (analytic functions with discrete poles) have this remarkable property. The procedure (14)–(15) therefore eliminates the real part of the cut-function (13) at the real axis $z = |z| \pm i\varepsilon = \beta_0 a(Q)/n \pm i\varepsilon$, the modified expression there is purely imaginary. The final result of the modified method is then just the Principal Value of the Borel integral, when $a(Q) > 0$

$$\Delta[z = \beta_0 a(Q)/n] = \text{Re} \Delta_{\text{BI}}(z \pm i\varepsilon) = \text{Re} \int_{\pm i\varepsilon}^{\infty \pm i\varepsilon} db \exp \left[-\frac{b}{\beta_0 a(Q)} \right] \frac{R(b)}{(1 - b/n)^{1+\nu}}, \quad (16)$$

and the new cut-function can be written for a general complex z as

$$\begin{aligned} \Delta_{\text{cut}}(z) = & +C \frac{n}{\beta_0} \frac{\pi}{\Gamma(1+\nu) \sin(\pi\nu)} [(-z)^{-\nu} - \cos(\pi\nu) z^{-\nu}] \\ & + C \frac{n}{\beta_0} \sum_{m=1}^{\infty} \kappa_m \frac{\pi}{\Gamma(1+\nu-m) \sin(\pi\nu)} [(-z)^{-\nu+m} - (-1)^m \cos(\pi\nu) z^{-\nu+m}]. \end{aligned} \quad (17)$$

It can be checked explicitly that this function is finite for any finite complex ν , including $\nu = 0, \pm 1, \pm 2, \dots$. It is an analytic function of z outside the real axis, with the cut along the real axis. The subtraction of the poles of $\cot(\pi\nu)$ (15) introduced in (17) terms proportional to $z^{-\nu}$ or $z^{-\nu+m}$. The consistency of the results for $\nu = 0, \pm 1, \dots$ thus introduced an additional cut along the negative real axis.

The central idea in Ref. [9] was that the cut-function to be subtracted from the Borel-resummed observables with an IR renormalon has a cut only along the positive part of the real axis. In the two exactly solvable non-QCD examples presented there this idea was shown to hold [9]. It remains unclear whether the cut-function (8) of the QCD prescription of Ref. [9] can be regularized with respect to ν in a physically tenable way and at the same time maintaining the cut only along the positive axis.

The IR renormalons play an important role also in the resummations using modified Borel transforms where the entire integrand in the Borel integration is renormalization scale (RS) invariant. Such transforms were introduced

by Grunberg [13] on the basis of a larger class of transforms proposed in Ref. [14] in a somewhat different context. Such RS-invariant Borel transform resummations were applied in Refs. [15, 16], by either evaluating the Principal Value of the Borel integral [15] or adding to the Principal Value the higher-twist OPE terms [16]. The discussed method of Ref. [9], for the ordinary Borel transforms (3), can be adapted to the method of the RS-invariant Borel transforms. The problems (divergences) appearing in this case are similar to those discussed here, but algebraically more complicated. It is not clear whether in such case an analogous regularization procedure as the one presented here would lead to the Principal Value of the RS-invariant Borel resummation.

In the present work, the method of subtracting a power-suppressed term from the Principal Value of the Borel integral for QCD observables with IR renormalon, recently proposed in Ref. [9] and applied in Refs. [10, 11], has been scrutinized. It was pointed out that the result becomes physically untenable for specific values of the renormalon power coefficient ν , as a consequence of the divergences of a factor $[\cot(\pi\nu)]$ appearing in the result. When these divergences are removed in apparently the most natural way, the power-suppressed term of the method disappears and the modified result becomes the Principal Value. These conclusions suggest that the most natural pQCD Borel integration of a QCD observable remains the Principal Value. The additional power-suppressed (higher-twist, higher-dimensional) terms cannot be inferred from pQCD (+renormalon) methods in any natural way. Such additional (OPE) terms involve vacuum expectation values (VEVs) of higher-twist operators and can theoretically be obtained or estimated only by genuinely nonperturbative methods. Phenomenologically, such additional OPE terms can be determined by fitting them to the corresponding experimental data. However, in such a procedure, it is important to keep for the leading-twist term in the OPE a specific pQCD expression, which should apparently most naturally be the Principal Value of the Borel integral, i.e., Eq. (16). On the other hand, if the leading-twist term is taken to be a truncated perturbation series (TPS), the strength of the higher-twist terms will sometimes dramatically change when the order of the TPS is changed [17, 18].

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